Full-Wave Analysis of Strip Transmission Line on Circular Dielectric Rod

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***Abstract*—Dispersion characteristics of the strip transmission line on a circular dielectric rod is investigated based on a full-wave analysis using the eigenfunction-weighted boundary integral equation method. Numerical results clearly demonstrate the usefulness of this method in finding propagation properties of the dominant mode and high-order modes, and variation of parameters with respect to the radius of the rod and the angle occupied by the strip conductor for a wide frequency range.**

I. Introduction

RECENTLY, several structures such as rectangular [1], trapezoidal [2], and valley microstrip lines [3] have been proposed for reducing conductor losses. The theoretical modeling of such complicated structures with curved interfaces still remains as a problem under development.

In the analysis of transmission lines with curved dielectric interfaces, the boundary integral equation method is attractive because of its freedom in treating various cross-sections. Charles *et al.* [4] have extended the boundary element method to analyze a dielectric waveguide with cross-sections of arbitrary shape. Kishi *et al.* [5] have presented a boundary integral method without using Green's function to analyze uniform-core optical fibers. The authors have also proposed an eigenfunction-weighted boundary integral equation method to analyze various planar transmission lines [6].

It is shown in this letter that propagation properties of the strip transmission line on a circular dielectric rod can be easily analyzed using the eigenfunction-weighted boundary integral equation method.

II. Formulation to Treat Arbitrary Cross-Sections

Fig. 1 shows the cross-section of a strip transmission line on a circular dielectric rod, which is made of two homogeneous media, that is, the air and dielectric region. Due to the symmetry of the configuration, only half section with an equivalent magnetic wall on the symmetrical plane has to be taken into account for the dominant mode analysis.

A pair of boundary integral equations defined on the interface between the two regions are initially set up with respect to the TM-mode and the TE-mode field function in each region based on the Green's identity of the second kind [6]. The normal derivatives of the field functions on the interface are transformed into their tangential components with relations derived from Maxwell's equations. Two integral equations defined on the interface including conductor surface and dielectric interface for the TM-mode and the TE-mode field function in a homogeneous region are written as follows:

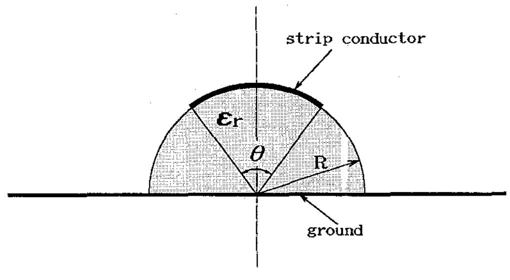

Fig. 1. Cross-section of the strip transmission line on a circular dielectric rod.

$$\int_\Gamma [W_e \frac{jk_c^2}{\omega\varepsilon} H_t - W_e \frac{\beta}{\omega\varepsilon}\frac{\partial H_z}{\partial t} - \frac{\partial W_e}{\partial n} E_z] d\Gamma = 0, \quad (1a)$$

$$\int_\Gamma [W_h \frac{jk_c^2}{\omega\mu} E_t - W_h \frac{\beta}{\omega\mu}\frac{\partial E_z}{\partial t} + \frac{\partial W_h}{\partial n} H_z] d\Gamma = 0, \quad (1b)$$

where k_c is the cutoff wavenumber. β denotes the unknown propagation constant. The symbol t and n denote the tangential and normal components to the interface, respectively. Γ denotes the whole interface contour between the two regions. W_e and W_h are two weighting functions corresponding to the TM-mode and the TE-mode, respectively. In general, the two weighting functions, W_e and W_h, are chosen from the Green's function [4]. However, these weighting functions can also be chosen from the eigenfunctions for the cylindrical coordinates [5]. The weighting functions in the dielectric region are then given by

$$W_e = J_i(kc_1 r)\sin(i\theta) \qquad (i = 1, 3, \ldots), \quad (2a)$$
$$W_h = J_i(kc_1 r)\cos(i\theta) \qquad (i = 1, 3, \ldots), \quad (2b)$$

where J_i denotes the ith-order Bessel function of the first kind. The weighting functions in the air region are also given by

$$W_e = K_i(kc_2 r)\sin(i\theta) \qquad (i = 1, 3, \ldots), \quad (3a)$$
$$W_h = K_i(kc_2 r)\cos(i\theta) \qquad (i = 1, 3, \ldots), \quad (3b)$$

where K_i denotes the ith-order modified Bessel function of the second kind.

A system of coupled boundary integral equations can be constructed from (1) after applying the conditions of the continuity of tangential fields to the dielectric interfaces and the extinction of tangential electrical fields on the strip conductor surfaces. In the numerical analysis, the contours of the

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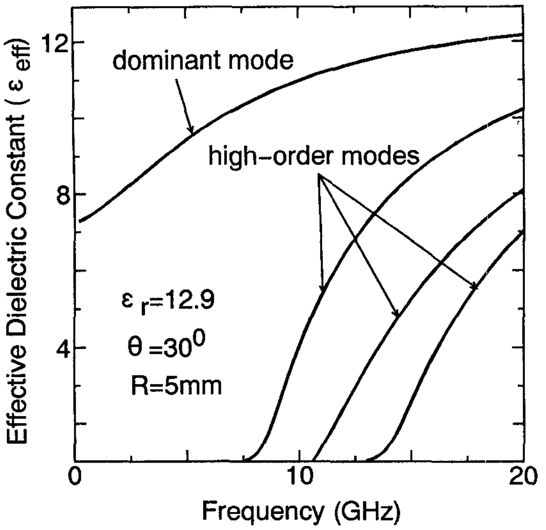

Fig. 2. Dispersion properties of the dominant mode and high-order modes.

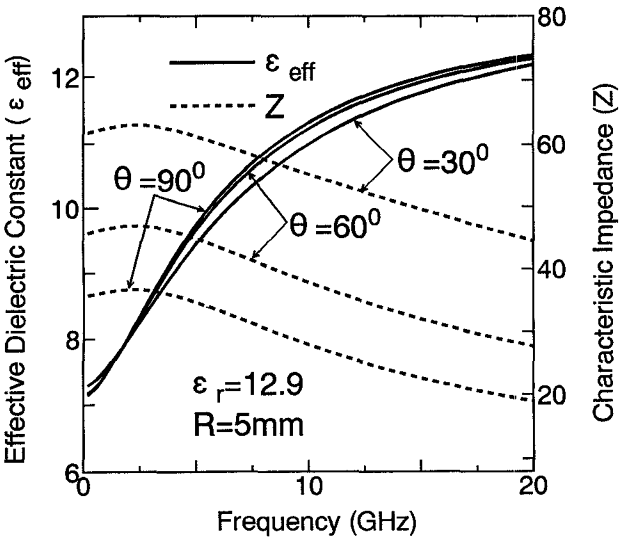

Fig. 3. Effective dielectric constants and characteristic impedances for different angles occupied by the strip conductor.

whole interface are divided into small segments, the number of which is adjusted to that of eigenfunctions. The effective dielectric constants are derived from the eigenvalues of these homogeneous linear equations. The characteristic impedance of the dominant mode is defined as the ratio of the voltage to the current of the transmission line.

III. Numerical Results

Fig. 2 shows the dispersion effects of the propagation constant of the dominant mode and high-order modes in the case of even symmetry. Fig. 3 and Fig. 4 show the effective dielectric constants and characteristic impedances for the dominant mode as functions of the angle occupied by the

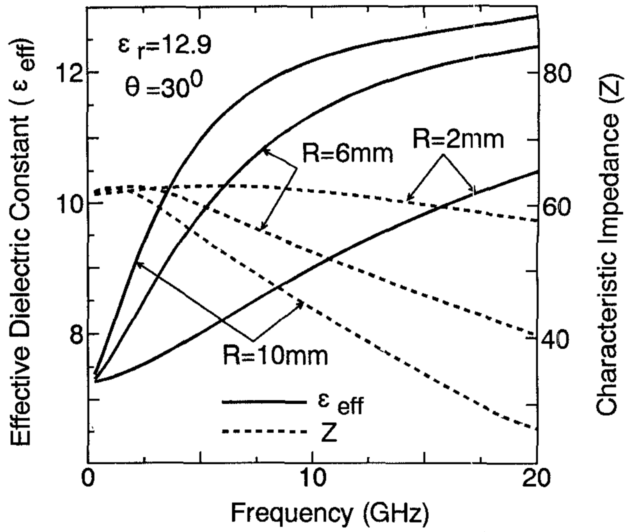

Fig. 4. Effective dielectric constants and characteristic impedances for different radii of the rod.

strip conductor and the radius of the circular rod, respectively. Fig. 3 shows that variation of effective dielectric constants with respect to frequencies is approximately invariable to the angle of the rod. On the other hand, it is also found in Fig. 4 that the characteristic impedances are almost independent of the radii in the case of a constant angle at low frequencies.

IV. Conclusion

The eigenfunction-weighted boundary integral equation method was extended to the full-wave analysis of the strip transmission line on a circular dielectric rod. Some numerical results were shown to clarify the propagation properties of this strip transmission line structure. The present method can be also applied to analyze other transmission lines on a dielectric rod of arbitrary cross-sections.

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